



Revising Measurement-Based Probabilistic Timing Analysis

RTAS 2017

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ONERA - The French Aerospace Lab

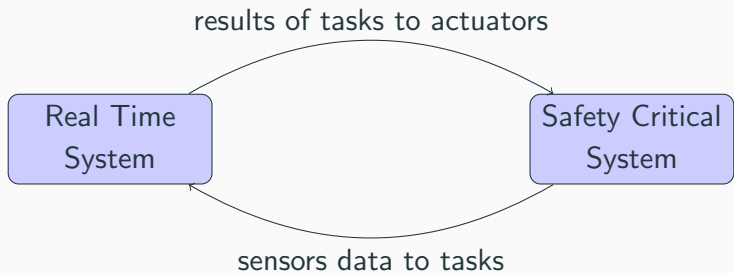
Agenda

1. Introduction
2. Background
3. EVT Applicability
4. MBPTA in Practice
5. Conclusion and Future Work

Introduction

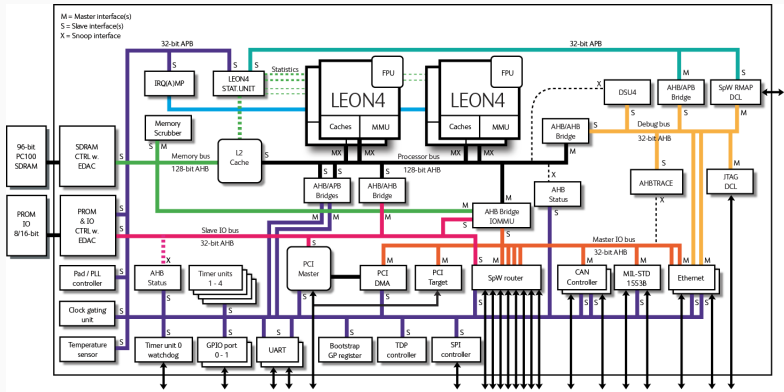
Introduction

- **Predictability** is of paramount importance in real time systems



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- **Predictability** is of paramount importance in real time systems
- Worst case properties are hard to guarantee due to today's systems complexity

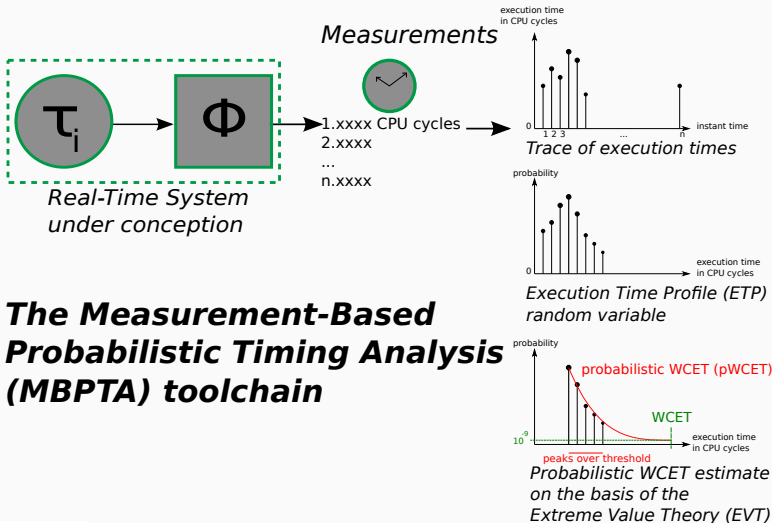
- Measurement-Based Probabilistic Timing Analysis (MBPTA) translates the Worst Case Execution Time (WCET) into the probabilistic WCET (pWCET)
- MBPTA makes use of the Extreme Value Theory (EVT) for inferring pWCETs from execution time measurements

Contributions

- Guidelines for a systematic and complete EVT application
- Algorithms for best estimating the pWCET parameters wrt measurements
- Quantification of the pWCET estimate reliability
- Application of the proposed approach to non-time randomized and time randomized systems

Background

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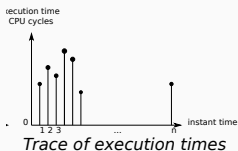


Background

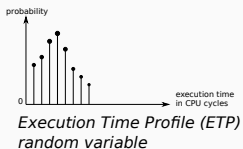
C_1, \dots, C_n a trace of **independent** and **identically distributed** (iid) execution time measurements from the ETP \mathcal{C} of cumulative distribution function $F_{\mathcal{C}}$, and we denote

$$F^u(c) = P\{\mathcal{C} \leq u + c \mid \mathcal{C} > u\},$$

the distribution of the peaks over the threshold u



(a) Trace



(b) \mathcal{C}

Background

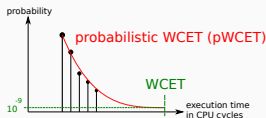
[Pickands theorem] F_C is in the **Maximum Domain of Attraction** of a Generalized Extreme Value distribution of shape parameter ξ iff

$$\lim_{u \rightarrow c_0} \sup_{0 \leq c \leq c_0 - u} |F^u(c) - GPD_\xi(c)| = 0,$$

with GPD_ξ the **Generalized Pareto Distribution (GPD)** defined as:

$$P(C \leq c) = GPD_\xi(c) = \begin{cases} 1 - (1 + \xi \times (c - u)/\alpha_u)^{-1/\xi} & \text{if } \xi \neq 0, \\ 1 - \exp(-(c - u)/\alpha_u) & \text{if } \xi = 0, \end{cases}$$

defined on $\{c, 1 + \xi(c - u)/\alpha_u > 0\}$



*Probabilistic WCET estimate
on the basis of the
Extreme Value Theory (EVT)*

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- $\xi < 0$ Weibull distribution
- $\xi = 0$ Gumbel distribution
- $\xi > 0$ Frechet distribution

Background

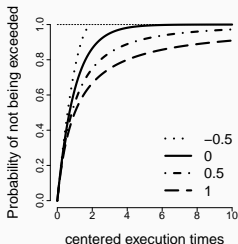
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EVT Applicability

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iid case H

stationary case H' (Generalized
EVT)

h_1 identically distributed from a
distribution

h'_1 stationarity

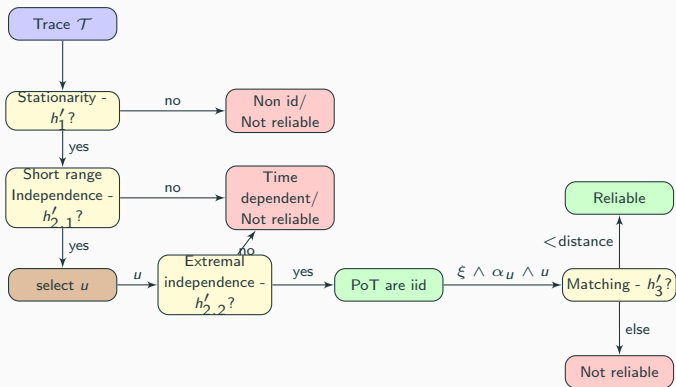
h_2 independent measurements

$h'_{2.1}$ short range dependence

h_3 in the Maximum Domain of
Attraction of a Generalized Extreme
Value distribution

$h'_{2.2}$ local independence of the peaks
 h'_3 experimental distribution of the
peaks matches the theoretical
distribution

EVT Applicability



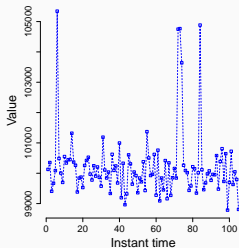
MBPTA decision diagram with tests and action applied.

EVT Applicability

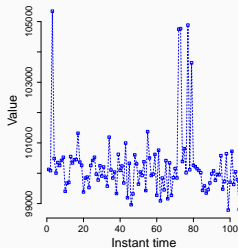
The pWCET estimate in case of extremal independence \bar{c}^{ei} is greater than or equal to the one in case of independence \bar{c}^i :

$$\bar{c}^{ei} \geq \bar{c}^i$$

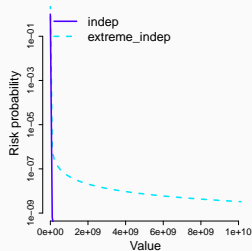
Dependent measurements provide more pessimistic pWCET estimates than the independence



(a) Extremal independence



(b) Artificially induced independence



(c) pWCETs

MBPTA in Practice

MBPTA in Practice

Industrial safety-critical embedded system *trace1*

Robotic embedded system *trace2, trace3*

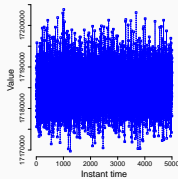
Multicore real time system *trace4 to trace7*

GPU CUDA system *trace8, trace9*

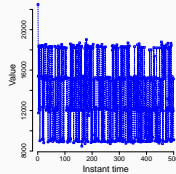
Artificial traces *trace10 to trace12*

FPGA real time system *trace13, trace14*

MBPTA in Practice

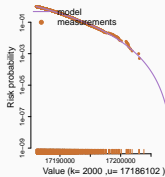


(d) *trace1*

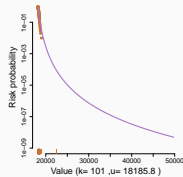


(e) *trace4*

Traces of executions times: variability of execution time with non-time randomized architectures.



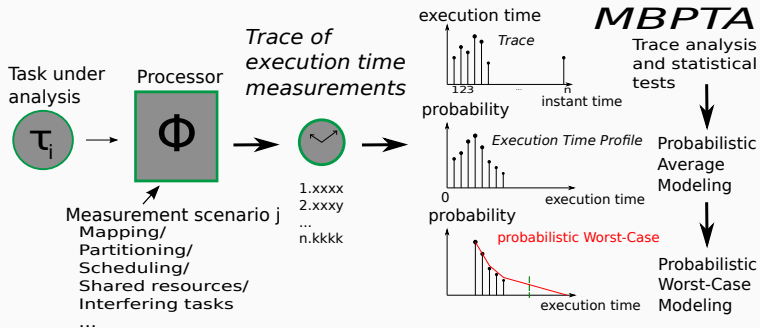
(a) *trace1*



(b) *trace4*

pWCET models from input traces. Execution times are in logarithmic scale.

MBPTA in Practice

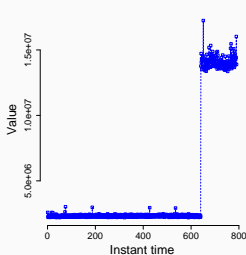


MBPTA in Practice

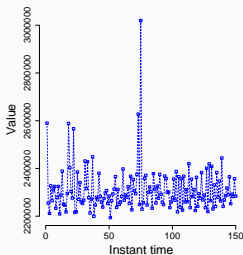
With J the finite set of possible measurement scenarios for the system

1. Trace-merging $\mathcal{T}_c, \mathcal{T}_c \stackrel{\text{def}}{=} \bigcup_{j \in J} \mathcal{T}_{c_j}$
2. Envelope pWCET model of \bar{C}^j for all $j \in J$. The worst-case distribution \bar{C} for every $j \in J$ is such that:

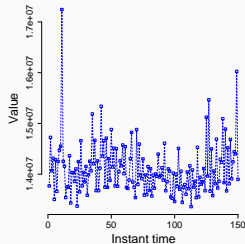
$$\bar{C} \stackrel{\text{def}}{=} \max_{j \in J} \{\bar{C}^j\}$$



(a) exploration



(b) exp1



(c) exp2

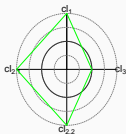
Multi-scenario trace and single-scenario traces.

MBPTA in Practice: EVT applicability

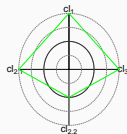
- *pWCET Reliability*: the *pWCET* estimate is safe if for all hypotheses in H' are verified

Each hypothesis h'_i is tested and returns a confidence level cl_i from 0 (low confidence) to 4 (high confidence) on the acceptance of h'_i

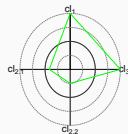
The set of hypotheses H' is verified if for all i $cl_i \geq 1$



(a) trace2



(b) trace3



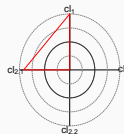
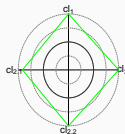
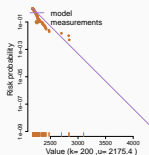
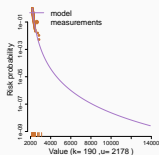
(c) trace4

- *pWCET accuracy*:

$$acc \stackrel{\text{def}}{=} \frac{\langle WCET; p \rangle - \max(\mathcal{T})}{\max(\mathcal{T})}$$

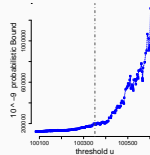
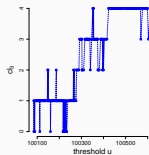
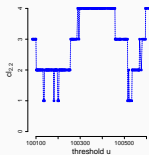
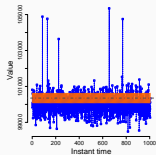
MBPTA in Practice: Best fitting algorithms

- *Gumbel model restriction*



(d) *trace6* (no Gumbel) (e) *trace6* – G (Gumbel) (f) *trace6* (no Gumbel) (g) *trace6* – G (Gumbel)

- *Threshold selection*



(h) Threshold selection to respect the WCET; 10^{-9} and threshold u (i) $c_{2.2}$ variation to respect the threshold u (j) c_3 maximization (k) $\langle WCET; 10^{-9} \rangle$ and threshold u

Conclusion and Future Work

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- Decision diagram for applying the EVT
- Algorithms for best fitting the execution time measurements
- Metrics for quantifying the pWCET estimate reliability and accuracy
- EVT diagnosed to be applicable to both time and non-time randomized systems
- Worst-case guarantees of the measurement scenarios

Ongoing work: What are the measurement scenarios for trustful pWCET estimates?

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Ongoing work: What are the measurement scenarios for trustful pWCET estimates?

- Contentions within the processor shared resources
- Task input parameters

Questions?